

Using **math** | **MATH** | **Frac** on the TI-84 family of calculators

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Once we acquire a skill and practice it, learning new skills sometimes begs a few shortcuts around skills we've already mastered. In particular, working with fractions - fractions with numbers, as opposed to variables - is one such area. The TI-84 family of calculators provides a few timesaving features for working with fractions. Let's explore some.

Question.

How can we take advantage of the fraction features of **math** | **MATH** | **Frac** on the TI-84 family of calculators?

Discussion.

The Texas Instruments TI-84 calculator appears to be a standard tool for high school math students and beyond. One of the best moves I have made as a pro math tutor and test-prep coach was to add the color version of this graphing calculator to my toolbox¹.

1 Precision, exact answers, and rounding

This topic is sometimes complicated, and mostly beyond the scope of this article, but it is relevant. A lot of my students are often perplexed by my practice of trying to keep fractions around (on paper) for as long as possible while doing algebra, leaving any simplifying only 'til the very end of a problem, even if the question asks for an answer in decimal form. The essence of my reasoning is this: **the earlier you switch to decimals (and, therefore, approximations), the more your answer deviates from the correct final answer in the end.** This is due mostly to the properties of rational and approximating irrational numbers, topics that you likely have only barely touched on in your Math classes.

1.1 Rational and irrational numbers

Recall:

Definition 1. A *rational number* is any number that can be expressed as a *ratio* (fancy way of saying *fraction*) of integers.

Typical characteristics of rational numbers are that they have a finite number of decimal places in decimal form, as well as those that repeat sequences of one or more digits.

¹One other great move I made, as many of my readers know, is to learn how to use Desmos (<http://desmos.com/calculator>).

Some examples, showing how ratios of integers play out in decimal form:

$$235 = \frac{235}{1} = 235 \quad (\text{any integer})$$

$$\frac{1}{3} = 0.3333333333\dots = 0.\overline{3} \quad (\text{infinitely repeats})$$

$$\frac{1}{9} = 0.125 \quad (\text{terminates})$$

$$\frac{448}{999} = 0.448448448448\dots = 0.\overline{448} \quad (\text{pattern 448 repeats infinitely})$$

Definition 2. *An irrational number is any number that cannot be described as a rational number.*

Typical characteristics of "irrational numbers, ... [in their decimal forms] do not terminate, nor do they repeat, i.e., do not contain a subsequence of digits, the repetition of which makes up the tail of the representation."²

Some examples:

$$\begin{aligned} \pi &= 3.14159265358979323846264338327950288419716939937510\dots \\ e &= 2.71828182845904523536028747135266249775724709369995\dots \\ \sqrt{2} &= 1.41421356237309504880168872420969807856967187537694\dots \\ &0.090090009000090000090000090000009000000090000000090000000000\dots \end{aligned} \quad (1)$$

As an aside, (1) is interesting. While we can see a pattern (number of zeroes is increasing by 1 after each 9), it is not a pattern that we can represent using a simple ratio of integers, therefore disqualifying it as a rational number.

It's worth it to note, that, oftentimes, combining rational and irrational numbers via basic arithmetic operations (adding, subtracting, multiplying, dividing), will result in an irrational number. This can serve as a *very rough* rule of thumb when trying to predict what the result of operations might be. However, note that sometimes, combining irrational numbers via arithmetic operations produces a rational number! Simple example: $(\sqrt{2})^2 = \sqrt{2} \cdot \sqrt{2} = 2$. Rational!

1.2 Precision

Knowing *when* to introduce the calculator and/or estimates into a problem's solution is a fine art. Here is a simple example done two ways:

$$\frac{1}{3}x - 680 = 40$$

$$\frac{1}{3}x = 720 \quad (\text{add 680 to both sides})$$

$$x = 2160 \quad (\text{multiply both sides by 3})$$

And

$$\frac{1}{3}x - 680 = 40$$

²https://en.wikipedia.org/wiki/Irrational_number

$$\frac{1}{3}x = 720 \quad (\text{add } 680 \text{ to both sides})$$

$$\begin{aligned} 0.33 \cdot x &= 720 && (\text{convert } 1/3 \text{ to decimal approximation}) \\ x &= 2181.818182 && (\text{divide both sides by } 0.33) \end{aligned} \quad (2)$$

This simple problem demonstrates the crux of the issue with decimal approximations of rational numbers, as:

$$2160 \neq 2181.818182$$

In fact, with only *one* approximation, we introduced a rather large error; this is the risk of *going to the calculator too soon*. The correct answer is indeed 2160. As an exercise to the reader, start with (2), above, and approximate $\frac{1}{3}$ with 0.333, then do it again with 0.3333, 0.33333 and so on. You should see the answer for each successive approximation of $\frac{1}{3}$ get closer and closer to the correct answer of 2160. Calculus students will recognize this process: calculating a limit by successive approximation.

2 How fractions work on the TI-84 family of calculators

The TI-84 family has the ability to work with rational numbers, in reduced fraction form, in the range of

$$-\frac{999997}{9999} \leq x \leq \frac{999997}{9999} \quad (3)$$

$$-100.009701 \leq x \leq 100.009701 \quad (4)$$

To further the point, using basic counting principles, there are $2 \cdot 999997 \cdot 9999 + 1$ rational numbers in this range - quite a lot of them.

3 Using **math** | **MATH** | **Frac**

3.1 Give it a try

On your calculator, enter: $4 \div 76$. Display says: **0.0526315789**. Now, hit **math**, then choose **Frac** from the **MATH** menu (usually already highlighted by default). The result should be $\frac{1}{19}$, which is the reduced form of $\frac{4}{76}$.

3.2 **math** | **MATH** | **Frac** is the best fraction reducer and more

To reduce any fraction that can be represented as a rational number on the interval mentioned earlier in this section, just do the division in the calculator, then hit **math** | **MATH** | **Frac**. Simple. If the fraction is outside the ranges indicated in (3) and (4), then the result of **math** | **MATH** | **Frac** will just repeat the current answer. In fact, even if you do operations with mixed denominators, **math** | **MATH** | **Frac** will deal with all of that and provide a reduced fraction result. Quite handy.

3.3 **math** | **MATH** | **Dec** is **math** | **MATH** | **Frac**'s complement

If a fraction is the current value (**Ans**) in the calculator, you can use **math** | **MATH** | **Dec** (usually choice #2) to go back to the decimal form.

4 Going deeper with the **math** | **FRAC** menu

There are a few other options for displaying and dealing with fractions on the TI-84. The **math** | **FRAC** menu has some more options.

4.1 1. **n/d**

This allows you to enter a fraction in $\left(\frac{\text{numerator}}{\text{denominator}}\right)$ form. The interface provides a blank fraction to fill in. Use the arrow keys to navigate. This is the best way to enter any fraction, improper or otherwise.

4.2 2. **Un/d**

This allows you to enter a fraction in $\left(\text{whole}\frac{\text{numerator}}{\text{denominator}}\right)$ form. The interface provides a blank fraction to fill in. Use the arrow keys to navigate. This is the best way to enter a mixed number.

4.3 3. **▶ F ◀ ▶ n/d**

This allows you to go back and forth between decimal (**F**loating point decimal) representation and **n/d** representation. This fraction form includes reduced fractions and improper fractions. Whatever form you start with, this choice converts it to the other. This is the same as what **math** | **MATH** | **Frac** and **math** | **MATH** | **Dec** do.

4.4 4. **▶ n/d ◀ ▶ Un/d**

This allows you to go back and forth between **n/d** and **Un/d** representations, that is, improper fraction and mixed-number format. Whatever form you start with, this choice converts it to the other.

5 Some ways to use the TI-84's fraction features

I will update this section, ongoing. Check mymathteacheristerrible.com for updates.

5.1 Trigonometry: radian measure

If you're studying trigonometry, you eventually have to deal with radian measures of angles. Radians are usually expressed by a quantity multiplied by π . Typical values we see are: $\frac{\pi}{2}$, $\frac{3}{4}\pi$, 2π , $\frac{11\pi}{6}$, etc. Often, when using the calculator, however, we get some result where it's difficult to tell whether our result is, in any part (usually by factoring), a rational number. With radians, in particular, π is irrational. Let's consider an example:

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 0.7853981634$$

math | **MATH** | **Frac** and any of its comparable functions discussed here just yield the same number, that is, the answer is either irrational or is outside the range discussed in (3) or (4). Not really helpful. But, if we divide our answer by π , what's left over has the potential to be the rational number we're multiplying by:

$$0.7853981634 \div \pi = 0.25$$

And, of course, if we hit **math** | **MATH** | **Frac**, we get $\frac{1}{4}$, so our angle, in radians, is $\frac{\pi}{4}$. Now, this example was pretty obvious, but it extends to more complicated values. Let's try:

$$\begin{aligned} \cos\left(\frac{11\pi}{6}\right) &= 0.8660254038 \\ \cos^{-1}(0.8660254038) &= 0.5235987756 \\ 0.5235987756 \div \pi &= 0.1666666667 \end{aligned}$$

If we hit **math** | **MATH** | **Frac** on this result, we get only $\frac{1}{6}$. Wait, what? Well, remember, for sine and cosine, there are usually *two* answers for any particular y -value. The calculator returns only the value of the *reference angle* for the y -value, so we must compute the other value. Specifically, since cosine is positive in the first quadrant and the fourth, a reference angle of $\frac{1}{6}$ rad also determines an angle of $-\frac{1}{6}$ rad, which is $2\pi - \frac{1}{6}\pi$, which is indeed $\frac{11\pi}{6}$. Tricky.

What a cool technique, though! The idea is that, by dividing out π , we potentially remove, i.e., factor out the irrationality, from our angle. We can then (hopefully) come up with the rational portion of the quantity, to express the angle as a ratio multiplied by π . A rational factor is not really guaranteed, though. For example, suppose we want to know more about $\sqrt{2}\pi$ radians? Right. Not going to be able to get there from the decimal representation (4.442882938...) using any of the techniques discussed here.

5.2 Problems that ask for exact value

Not so much a cool technique, rather, this is a more precise definition of terms. I'd say you should use **math** | **MATH** | **Frac** anytime a question asks for exact answers, with the caveat that it only works if the exact answer is a rational number. When a problem asks for an exact value, it's a bit of a signal. It's really asking, *What is the most compact way to represent the answer without compromising precision?* Fact is, *every* decimal representation of a non-terminating decimal is an approximation, plain and simple. Typical examples of *exact answers* are things like $\frac{\sqrt{2}}{2}$, $\frac{\pi}{3}$, and $\frac{1}{3}$.

This is the major reason why I elect to keep fractions around for as long as possible while solving a problem. Furthermore, I often opt to give answers in rational form, regardless of whether the question asks for one, as the rational number form is *always* the exact answer.

5.3 Checking to see if a value is rational, i.e., confirming/denying intuition

Example. Is $\frac{5.\overline{3}}{30}$ rational?

Discussion. It appears that we have some rational components of this particular fraction. In the numerator, we have $5.\overline{3}$, which simply is $5 + \frac{1}{3}$. Certainly the denominator, 30, is rational. We would enter into the calculator as:

$$(5 + 1/3) \div 30$$

Answer is: $.1777777778$; this appears to be a repeating decimal. **math** | **MATH** | **Frac** gives $\frac{8}{45}$. Rational!

Done.

Reporting errors and giving feedback

I am so pleased that you have downloaded this study guide and have considered the techniques herein. To that end, I am the only writer and the only editor of these things, so if you find an error in the text or calculations, please email me and tell me about it! I am committed to prompt changes when something is inaccurate. I also really appreciate it when someone takes a moment to tell me how I'm doing with these sorts of things, so please do so, if you feel inclined.

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